MATHEMATICS 31

Mr. M Cherney

COURSE OUTLINE 2024-2025

Ch 1 Algebra	9(10 FR)-8 Classes	11(12) School Days	Sept 3 – Oct 1
Ch 2 Limits	6-5 Classes	7 School Days	Oct 3 – Oct 21
Ch 3 Derivatives	8-7 Classes	10 School Days	Oct 22 – Nov 14
Ch 4 Advanced Derivatives	8-5 Classes	10 School Days	Nov 18 – Dec 9
Ch 5 Curve Sketching	10-7 Classes	14 School Days	Dec 10 – Feb 4
Ch 6 Derivative Applications	7(8 OE)-6 Classes	9(10) School Days	Feb 6 – Mar 6
Ch 7 Antiderivatives	9-8 Classes	11 School Days	Mar 10 – Apr 15
Ch 8 Integration Applications	5-5 Classes	6 School Days	Apr 17 – May 1
Course Review/Final	12(14 OE)-12 Classes	15(17) School Days	May 5 – June 13
	74(63) Classes	93(97) School Days	

Final

Final Exam	May 5 – June 13

COURSE MARKING 2024-2025

Heading	Date	Weight	Points Earned (%)	Percent (%)
Course Work		70		
Tests		90		
Chapter 1 Algebra		10		
Chapter 2 Limits		10		
Chapter 3 Derivatives		15		
Chapter 4 Advanced Derivatives		10		
Chapter 5 Curve Sketching		15		
Chapter 6 Derivative Applications		15		
Chapter 7 Antiderivatives		15		
Chapter 8 Integration Applications		10		
Homework		10		
Final Exam		30		
Final Grade				

Daily Homework for each assignment is due the day after it is assigned. It will be marked for completeness, 1 mark for each completed question out of the total assigned questions. Incorrect questions should be corrected.

Review Summary Sheets are given for each chapter and can be used as 'I Can' statements to self assess learning or as review sheets for content covered in the chapter.

Tests may be rewritten on any chapter once at any time during the year before the beginning of the Course Review at the end of the year. Your best score up to 79% will be taken on rewrites. Before any test is rewritten all previous tests from other chapters must be complete and at least some homework from the rewritten chapter must be handed in.

Extra Help or a quiet place to work is available during any lunch hour in my room through out the year on a come and go as you need help basis.

Extra Practice

From Text Book

Chapter 1	Lesson 2	Pg 66 Question 1
		Pg 67 Questions 1 – 2
	Lesson 3	Pg 206 Questions 1 – 2
	Lesson 4	Pg 163 Questions 1 – 2
		Pg 166 Questions 1 – 2
	Lesson 5	Pg 116 Questions 1 – 2
	Lesson 8	Pg 267 – 269 Questions 1 – 3, 5
		Pg 273 – 275 Questions 2, 3, 5, 6, 7, 8, 10, 11
		Pg 279 – 280 Questions 5, 7, 8, 10, 11
		Pg 284 – 287 Questions 1, 3, 7, 11, 13, 15, 19, 20, 23, 27, 29, 35, 40, 45, 56
		Pg 291 – 292 Questions 1 – 4
Chapter 2	Lesson 1	Pg 56 – 57 Questions 1 – 5
		Pg 50 – 51 Questions 1 – 6, 8
	Lesson 2	Pg 18 – 20 Questions 1 – 6
		Pg 27 – 29 Questions 1 – 4, 6, 8, 12
	Lesson 4	Pg 9 – 10 Questions 7 – 11
		Pg 35 Question 7
		Pg 43 – 44 Questions 1, 3, 8
Chapter 3	Lesson 1	Pg 75 – 77 Questions 1 – 7, 9, 11
	Lesson 2	Pg 83 – 84 Questions 1 – 4, 7 – 9
	Lesson 3	Pg 88 – 89 Questions 1, 4, 6 – 9
		Pg 92 – 93 Questions 2, 6 – 8
	Lesson 4	Pg 95 – 96 Questions 2, 4, 7, 8
	Lesson 5	Pg 102 – 103 Questions 1, 3 – 5, 7, 9, 10
	Lesson 6	Pg 107 – 108 Questions 1 – 3
	Lesson 7	Pg 111 Questions 1 – 7
Chapter 4	Lesson 1	Pg 306 – 307 Questions 1 – 38 even
	Lesson 2	Pg 313 – 315 Questions 1 – 5, 7, 8
	Lesson 3	Pg 319 – 320 Questions 1 – 3, 9
	Lesson 5	Pg 361 – 362 Questions 4, 5, 7
		Pg 366 – 368 Questions 1, 3 – 11
		Pg 375 – 376 Questions 3 – 10, 17, 18
		Pg 383 – 384 Questions 1 – 8

Chapter 5	Lesson 2	Pg 170 – 171 Questions 1 – 5
		Pg 176 – 177 Questions 1, 2, 4 – 8
		Pg 182 Questions 1, 3, 4, 6
	Lesson 3	Pg 229 – 230 Questions 1 – 3, 6 – 8
		Pg 240 Questions 1 – 3, 7, 9, 12
	Lesson 4	Pg 232 Questions 1 – 3
		Pg 188 – 191 Questions 1 – 20 odd
		Pg 212 – 213 Questions 1 – 5
	Lesson 5	Pg 222 – 223 Questions 1, 2, 4 – 6, 8 – 11
	Lesson 6	Pg 244 Questions 1 – 3
Chapter 6	Lesson 1	Pg 188 – 191 Questions 1 – 20 odd
	Lesson 2	Pg 124 – 126 Questions 1 – 9
		Pg 128 – 129 Questions 1 – 9
	Lesson 3	Pg 134 – 135 Questions 1 – 9
		Pg 145 – 146 Questions 1 – 17
	Lesson 4	Pg 139 – 140 Questions 1 – 6
		Pg 195 – 196 Questions 1 – 8
	Lesson 5	Pg 390 – 392 Questions 1 – 14 even
	Lesson 6	Pg 152 Questions 1 – 8
Chapter 7	Lesson 1	Pg 408 – 409 Questions 1 – 5
	Lesson 3	Pg 411 – 412 Questions 1, 2, 3, 5, 6
		Pg 415 Questions 1 – 5
		Pg 420 – 421 Questions 1 – 7
		Pg 437 – 438 Questions 1, 2, 3, 7
	Lesson 4	Pg 454 – 455 Questions 1 – 3
		Pg 465 – 466 Questions 1 – 4
	Lesson 5	Pg 500 Questions 2 – 5
		Pg 505 – 506 Questions 1 – 4
	Lesson 7	Pg 461 Question 1
Chapter 8	Lesson 1	Pg 474 – 475 Questions 1 – 3
		Pg 500 Questions 2 – 5
	Lesson 2	Pg 511 – 512 Questions 3, 4

CALCULUS FORMULA SHEET

Graphing Calculator Window Format $x[x_{\min}, x_{\max}, x_{scl}]$ $y[y_{\min}, y_{\max}, y_{scl}]$

Properties of Limits

$$\lim_{x\to a}[f(x)+g(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$$

$$\lim_{x\to a}[f(x)-g(x)] = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$$

$$\lim_{x\to a}[cf(x)] = c\lim_{x\to a} f(x)$$

$$\lim_{x\to a}[f(x)g(x)] = \lim_{x\to a} f(x)\lim_{x\to a} g(x)$$

$$\lim_{x\to a}\left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$$

$$\lim_{x\to a}[f(x)]^n = \left[\lim_{x\to a} f(x)\right]^n, n>0$$

$$\lim_{x\to a}\sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)}$$

$$\lim_{x\to a} c = c \qquad \lim_{x\to a} x = a$$

$$\lim_{x\to a} x^n = a^n \qquad \lim_{x\to a}\sqrt[n]{x} = \sqrt[n]{a}$$

$$\lim_{x\to a} P(x) = P(a)$$

$$\lim_{x\to a}\frac{P(x)}{Q(x)} = \frac{P(a)}{Q(a)}$$

if $\lim_{x\to a^-} f(x) \neq \lim_{x\to a^+} f(x)$,

then $\lim_{x\to a} f(x)$ does not exist

if $\lim_{x\to a^-} f(x) = L = \lim_{x\to a^+} f(x)$,

then $\lim_{x\to a} f(x) = L$

$$\lim_{n\to\infty}\left(\frac{1}{n}\right)^r = 0,\ r>0$$

$$\lim_{n\to\infty} r^n = 0,\ |r|<1$$

$$\lim_{n\to\infty} S_n = \frac{a}{1-r},\ |r|<1$$

Properties and Rules of Derivatives

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$\frac{d}{dx}(c) = 0$$

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}f(x)$$

$$\frac{d}{dx}[f(x)+g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

$$\frac{d}{dx}[f(x)-g(x)] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$$

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}g(x) + g(x)\frac{d}{dx}f(x)$$

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}f(x) - f(x)\frac{d}{dx}g(x)}{[g(x)]^2}$$

$$\frac{d}{dx}[f(g(x))] = f'(g(x))\frac{d}{dx}g(x)$$

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}[g(x)]^n = n[g(x)]^{n-1}g'(x)$$

$$s = f(t)$$

$$v = \frac{ds}{dt} = f'(t)$$

$$a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = f''(t)$$

Newton's Method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Curve Sketching

if $f'(x) > 0$ on I, then f is increasing on I
if $f'(x) < 0$ on I, then f is decreasing on I

$f'(x) = 0$ is a local max or min or critical number

$\lim_{x \to a} f(x) = \pm\infty$ is a vertical asymptote
$\lim_{x \to \pm\infty} f(x) = L$ is a horizontal asymptote

if $f''(x) > 0$ on I,
then f is concave up on I

if $f''(x) < 0$ on I,
then f is concave down on I

if $f''(x) = 0$ on I,
then x is a possible inflection point

if $f'(c) = 0$ and $f''(c) > 0$,
then c is a local minimum

if $f'(c) = 0$ and $f''(c) < 0$,
then c is a local maximum

Slant asymptotes are found by division

$x -$ intercepts $(x, 0)$

$y -$ intercepts $(0, y)$

DR $M_2 I_2$ *ACIDS*

- Domain and Range
- Maximums
- Minimums
- Intercepts
- Inflection
- Asymptotes
- Concavity
- Increasing
- Decreasing
- Symmetry

Algebraic Relations

$$a^2 - b^2 = (a-b)(a+b)$$
$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$
$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

sigma properties

$$\sum_{i=1}^{n} c = c + c + c + \ldots + c = nc$$
$$\sum_{i=1}^{n} ct_i = c\sum_{i=1}^{n} t_i$$
$$\sum_{i=1}^{n} (t_i + s_i) = \sum_{i=1}^{n} t_i + \sum_{i=1}^{n} s_i$$

arithmetic series $t_n = a + (n-1)d$

$$S_n = a + (a+d) + (a+2d) + \ldots + [a + (n-1)d]$$
$$= \frac{n}{2}\left(2a + (n-1)d\right) = \frac{n(a + t_n)}{2}$$

geometric series $t_n = ar^{n-1}$

$$S_n = a + ar + ar^2 + \ldots + ar^{n-1}$$
$$= \frac{a(r^n - 1)}{r-1} = \frac{rt_n - a}{r-1}$$
$$S = \frac{a}{1-r}, \quad |r| < 1$$

natural numbers

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

natural number squares

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

natural number cubes

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

Trigonometric Identities

Reciprocal Identities

$$\csc x = \frac{1}{\sin x} \quad \sec x = \frac{1}{\cos x} \quad \cot x = \frac{1}{\tan x}$$

Quotient Identities

$$\tan x = \frac{\sin x}{\cos x} \quad \cot x = \frac{\cos x}{\sin x}$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$
$$\sec^2 x = 1 + \tan^2 x$$
$$\csc^2 x = 1 + \cot^2 x$$

Sum and Difference Identities

$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$
$$\sin(x-y) = \sin x \cos y - \cos x \sin y$$
$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$
$$\cos(x-y) = \cos x \cos y + \sin x \sin y$$
$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$
$$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

Double Angle Identities

$$\sin 2x = 2\sin x \cos x$$
$$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$
$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

Half Angle Identities

$$\cos\frac{x}{2} = \pm\sqrt{\frac{1}{2} + \frac{\cos x}{2}}$$
$$\sin\frac{x}{2} = \pm\sqrt{\frac{1}{2} - \frac{\cos x}{2}}$$

Related Angle Identities

$$\sin(\pi - x) = \sin x \qquad \sin(2\pi - x) = -\sin x$$
$$\cos(\pi - x) = -\cos x \qquad \cos(2\pi - x) = \cos x$$
$$\tan(\pi - x) = -\tan x \qquad \tan(2\pi - x) = -\tan x$$

$$\sin(\pi + x) = -\sin x \qquad \sin(-x) = -\sin x$$
$$\cos(\pi + x) = -\cos x \qquad \cos(-x) = \cos x$$
$$\tan(\pi + x) = \tan x \qquad \tan(-x) = -\tan x$$

Corelated Angle Identities

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x \qquad \sin\left(\frac{3\pi}{2} - x\right) = -\cos x$$
$$\cos\left(\frac{\pi}{2} - x\right) = \sin x \qquad \cos\left(\frac{3\pi}{2} - x\right) = -\sin x$$
$$\tan\left(\frac{\pi}{2} - x\right) = \cot x \qquad \tan\left(\frac{3\pi}{2} - x\right) = \cot x$$

$$\sin\left(\frac{\pi}{2} + x\right) = \cos x \qquad \sin\left(\frac{3\pi}{2} + x\right) = -\cos x$$
$$\cos\left(\frac{\pi}{2} + x\right) = -\sin x \qquad \cos\left(\frac{3\pi}{2} + x\right) = \sin x$$
$$\tan\left(\frac{\pi}{2} + x\right) = -\cot x \qquad \tan\left(\frac{3\pi}{2} + x\right) = -\cot x$$

Limits of Trigonometric Functions

$$\lim_{\theta\to 0}\sin\theta = 0$$

$$\lim_{\theta\to 0}\cos\theta = 1$$

$$\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1 \qquad \lim_{\theta\to 0}\frac{\theta}{\sin\theta} = 1$$

$$\lim_{\theta\to 0}\frac{\cos\theta - 1}{\theta} = 0 \qquad \lim_{\theta\to 0}\frac{1-\cos\theta}{\theta} = 0$$

Derivatives of Trigonometric Functions

$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\tan x = \sec^2 x$$

$$\frac{d}{dx}\csc x = -\csc x\cot x$$

$$\frac{d}{dx}\sec x = \sec x\tan x$$

$$\frac{d}{dx}\cot x = -\csc^2 x$$

Derivative Applications in Economics

$C(x) = \text{Cost Function}$

$C'(x) = \text{Marginal Cost Function}$

$p(x) = \text{Price or Demand Function}$

$R(x) = xp(x) = \text{Revenue Function}$

$R'(x) = \text{Marginal Revenue Function}$

$P(x) = R(x) - C(x) = \text{Profit Function}$

$C'(x) = R'(x) = \text{Maximum Profit}$

$c(x) = \dfrac{C(x)}{x} = \text{Ave Cost Function}$

$C'(x) = \dfrac{C(x)}{x} = \text{Average Cost Min}$

Limits/Derivatives of Exponential Functions

$$y = b^x$$

$$y' = \lim_{h\to 0}\frac{b^{x+h} - b^x}{h} = b^x \lim_{h\to 0}\frac{b^h - 1}{h}$$

$$\lim_{h\to 0}\frac{e^h - 1}{h} = 1$$

$$\lim_{x\to 0}(1+x)^{\frac{1}{x}} = e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$$

$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}e^u = e^u\frac{du}{dx}$$

$$\frac{d}{dx}e^{g(x)} = e^{g(x)}g'(x)$$

$$\frac{d}{dx}b^x = b^x\ln b \qquad \frac{d}{dx}b^u = b^u\ln b\frac{du}{dx}$$

Exponential Growth and Decay

rate is proportional to amount

$\dfrac{dy}{dt} = ky$ is solved (k) with $y = y_0e^{kt}$

Compound Interest

$$A = Pe^{rt}$$

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

Limits/Derivatives of Logarithmic Functions

$$y = \ln x$$

$$\frac{d}{dx}\ln x = \frac{1}{x}$$

$$\frac{d}{dx}\ln u = \frac{1}{u}\frac{du}{dx}$$

$$\frac{d}{dx}\log_b x = \frac{1}{x\ln b}$$

Differential Equations

If $y'' + ky = 0$, then
$y = A\cos\left(\sqrt{k}x\right) + B\sin\left(\sqrt{k}x\right)$

Hooke's Law

$$s'' + \frac{k}{m}s = 0 \quad \frac{d^2s}{dt^2} + \frac{k}{m}s = 0 \quad F = ks$$

$$s = A\cos\sqrt{\frac{k}{m}}\ t + B\sin\sqrt{\frac{k}{m}}\ t$$

Area

If $y = f(x)$, and $f(x) > 0$, then
$A(b) = F(b) - F(a)$
is the area under $f(x)$ from a to b

If $y = \frac{1}{x}$, and $x > 1$, then
$\ln x$ is the area under y from 1 to x

If $y = \frac{1}{x}$, and $0 < x < 1$, then
$-\ln x$ is the area under y from x to 1

Trapezoid Rule

$$A = \tfrac{b-a}{2n}\left(f(a) + 2f(x_1) + \ldots 2f(x_{n-1}) + f(b)\right)$$

Integrals and Antiderivatives Definitions

Sum of Infinite Rectangles

$$\int_a^b f(x)dx = \lim_{\Delta x \to 0} \sum_{i=a}^{b} f(x_i)\Delta x$$

$$\text{where } \Delta x = \frac{b-a}{n} \text{ and } x_i = a + i\Delta x$$

Fundamental Theorem of Calculus

$$\int_a^b f(x)dx = F(b) - F(a)$$

Integrals and Antiderivatives

General Rules

$$\int cf(x)dx = c\int f(x)dx$$

$$\int [f(x) + g(x)]dx = \int f(x)dx + \int g(x)dx$$

$$\int 0dx = C$$

$$\int 1dx = x + C \qquad \int adx = ax + C$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \text{ when } n \neq -1$$

$$\int ax^n dx = \frac{ax^{n+1}}{n+1} + C, \text{ when } n \neq -1$$

$$\int \frac{1}{x}dx = \ln|x| + C \qquad \int \frac{1}{x\ln b}dx = \log_b\left(|x|\right) + C$$

$$\int \sin xdx = -\cos x + C$$

$$\int \cos xdx = \sin x + C$$

$$\int \sec^2 xdx = \tan x + C$$

$$\int \csc^2 xdx = -\cot x + C$$

$$\int \sec x\tan xdx = \sec x + C$$

$$\int \csc x\cot xdx = -\csc x + C$$

$$\int e^x dx = e^x + C \qquad \int b^x \ln bdx = b^x + C$$

$$\int a^x dx = \frac{a^x}{\ln a} + C, \text{ where } a \neq 1$$

$$\int \frac{1}{x^2+1}dx = Tan^{-1}x + C$$

$$\int \frac{1}{\sqrt{1-x^2}}dx = Sin^{-1}x + C$$

Substitution Rule

$$\int_a^b f\left(g(x)\right)g'(x)dx = \int_{g(a)}^{g(b)} f(u)du, \text{ where } u = g(x)$$

Integration Techniques

Fundamental Theorem of Calculus

$$\int_a^b f(x)dx = F(b) - F(a)$$

Substitution

$$\int_a^b f(g(x))g'(x)dx = \int_{g(a)}^{g(b)} f(u)du,$$

where $u = g(x)$ and $\frac{du}{dx} = g'(x)$

or

$$\int F'(g(x))g'(x)dx = F(g(x)) + C$$

where $\frac{d}{dx} F(g(x)) = F'(g(x))g'(x)$

Partial Fractions

$$\int \frac{A(x)}{B(x)} dx = \int Q(x)dx + \int \frac{R(x)}{B(x)} dx$$

$$\int \frac{1}{\text{LIN}} dx, \quad \text{use normal methods}$$

$$\int \frac{A(x)}{B(x)} dx = \int \frac{A}{\text{LIN}} + \frac{B}{\text{LIN}} dx$$

$$\int \frac{A(x)}{(B(x))^n} dx = \int \frac{A}{B(x)} + \frac{B}{(B(x))^2} + \ldots dx$$

$$\int \frac{A(x)}{B(x)} dx = \int \frac{Ax+B}{\text{QUAD}} + \frac{C}{\text{LIN}} dx$$

Integration by Parts

$$\int u dv = uv - \int v du$$

$$\int f(x)g'(x)dx$$

$$= f(x)g(x) - \int g(x) f'(x)dx$$

Volume of Rotation

$$V = \pi \int_a^b (f(x))^2 dx$$

Average Value of a Function

$$\text{Average Value} = \frac{\int_a^b f(x)dx}{(b-a)}$$